UNCERTAINTY

NETSCORE-21 Meeting

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Outline

• Introduction
• Basic Example
• A Simple Stochastic Programming Approach
• An Stochastic Generation Expansion Planning Application
Introduction

• When facing realistic problems, we naturally incur in errors, or at least, in assumptions in data: Uncertainty
• What if the real situation does not behave as we thought?
• Maybe solving a problem needs us to have an “ace in the hole” to overcome unexpected fluctuations of some parameters
• Usually, some sensitivity analysis is done
• Also, obtaining different responses which depends on different credible scenarios
• In finance and planning problems, players want to hedge their investment, they need to recover the investment, and minimize the risk
Introduction

• Uncertainty in NETSCORE-21:
  ✓ Investment, and O&M costs
  ✓ Fuel: costs and availability
    ➢ Fuel markets? Prices subject to supply and demand?
    ➢ Interdependencies with transportation systems?
    ➢ How much of each fuel do we have “available” for using?
  ✓ Demand
    ➢ How much will it likely to increase?
  ✓ Identify uncertainties in other areas
    ➢ Uncertainties in transportation systems (demand, flows, new technologies), new generation technologies, sustainability issues (like emissions taxation), resiliency (linked to random events), etc.
  ✓ Other kinds of uncertainties
    ➢ Regulation, investment risks, market prices, ...
Potential tools

• Uncertainty and Optimization:
  ✓ How to deal efficiently with uncertainty in multistage optimization problems?
    - Scenario Analysis?
    - Sensitivity Analysis?
    - Stochastic Programming?
    - Optimal Control Theory?
    - Robust Control Theory?
    - Robust Optimization? (interesting ideas)
    - Something else?
A Basic Illustration

\[ \min_{x_1, x_2} x_1 + x_2 \]

\[ \text{s.t. } \omega_1 x_1 + x_2 \geq 7 \]

\[ \omega_2 x_1 + x_2 \geq 4 \]

\[ x_1, x_2 \geq 0 \]

\[ \omega_1 \in U[4,1] \]

\[ \omega_2 \in U[3,1] \]

\( \omega_1 \) and \( \omega_2 \) are random parameters

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( x_1^* )</th>
<th>( x_2^* )</th>
<th>Objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean values</td>
<td>2.5</td>
<td>0.667</td>
<td>1.6364</td>
<td>2.9091</td>
<td>4.5455</td>
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<tr>
<td>Worst values</td>
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<td>2.0076</td>
<td>4.9924</td>
<td>7.0000</td>
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<tr>
<td>Optimistic values</td>
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<td>1</td>
<td>1.8796</td>
<td>2.1204</td>
<td>4.0000</td>
</tr>
</tbody>
</table>

Expected Objective function

5.1818
A basic illustration

By considering all of the three scenarios in the same problem: Deterministic Equivalent

\[
\begin{align*}
\min_{x_1,x_2} & \quad x_1 + x_2 \\
\text{s.t.} & \quad x_1 + x_2 \geq 7 \\
& \quad 1/3 x_1 + x_2 \geq 4 \\
& \quad 5/2 x_1 + x_2 \geq 7 \\
& \quad 2/3 x_1 + x_2 \geq 4 \\
& \quad 4x_1 + x_2 \geq 7 \\
& \quad x_1 + x_2 \geq 4 \\
& \quad x_1, x_2 \geq 0
\end{align*}
\]

\[
x_1^* = 2.2125 \\
x_2^* = 4.7845 \\
\text{Obj. function} = 7.0
\]

Expected Value of Perfect Information
EVPI = 7.0 - 5.1818 = 1.8182

That’s the cost we have to cover due to not knowing the right data (uncertainty)
A basic illustration

We assumed a discrete distribution for $\omega_1$ and $\omega_2$; their distribution actually is continuous (uniform). So,

$$\begin{align*}
\min_{x_1, x_2} & \quad x_1 + x_2 \\
\text{s.t.} & \quad \omega_{1,s} x_1 + x_2 \geq 7, \quad \forall s \in S \\
& \quad \omega_{2,s} x_1 + x_2 \geq 4, \quad \forall s \in S \\
& \quad x_1, x_2 \geq 0
\end{align*}$$

Solving the LP using 200 random scenarios with the same probability:

$$\begin{align*}
x_1^* &= 4.4501 \\
x_2^* &= 2.5046 \\
\text{Obj. function} &= 6.955
\end{align*}$$
A basic illustration

Geometrically,
Another basic illustration

Consider the following linear program:

\[
\begin{align*}
\max_{x_1, x_2} & \ 2x_1 + 3x_2 \\
\text{s.t.} & \ 2x_1 + 3x_2 \geq 2 \\
& \ 4x_1 + 3x_2 \leq 3/2 \\
& \ x_1, x_2 \geq 0 \\
& \ \omega_1 \in U[3,4] \\
& \ \omega_2 \in U[8,3/2] 
\end{align*}
\]

<table>
<thead>
<tr>
<th>Scenario</th>
<th>(w_1)</th>
<th>(w_2)</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>Objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean values</td>
<td>2.667</td>
<td>0.9375</td>
<td>0.2892</td>
<td>1.2289</td>
<td>1.2289</td>
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<tr>
<td>Largest values</td>
<td>4</td>
<td>1.5</td>
<td>0.2</td>
<td>1.2</td>
<td>1.2000</td>
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<tr>
<td>Least values</td>
<td>1.333</td>
<td>0.375</td>
<td>0.5217</td>
<td>1.3043</td>
<td>1.3043</td>
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<tr>
<td>Worst Values</td>
<td>1.333</td>
<td>1.5</td>
<td>Infeasible</td>
<td></td>
<td>Minus infinite</td>
</tr>
<tr>
<td>Mixed Values</td>
<td>4</td>
<td>0.375</td>
<td>0.1379</td>
<td>1.4483</td>
<td>1.4483</td>
</tr>
</tbody>
</table>

The approach considered is not general. How to deal with infeasibilities?
Recourse Function

Therefore, considering all the scenarios in the same LP results in an infeasible problem. The Recourse Function allows us to handle that situation

**General problem**

\[
\begin{align*}
\min_{x} & \quad c^T x \\
\text{s.t.} & \quad A \phi^T x \geq b \phi^T \\
x & \in X
\end{align*}
\]

**Deterministic Equivalent DE**

\[
\begin{align*}
\min_{x} & \quad z = c^T x + E_{\omega} \left[ \min_{y} q \phi^T y \left| Wy \geq b \phi^T - A \phi^T x, \ y \geq 0 \right. \right] = c^T x + Q \phi^T \\
\text{s.t.} & \quad x \in X
\end{align*}
\]

\[x: \text{first-stage variables}\]
\[y: \text{second-stage variables}\]
Recourse Function

Using the concept of the Recourse function, we get the DE for our problem

\[
\max_{x_1, x_2} z = x_2 + Q \xi_1, x_2
\]

s.t. \( x_1, x_2 \geq 0 \)

Where

\[
Q \xi_1, x_2 \equiv \min \left\{ \sum_{s \in S} p_s \xi_{1,s} y_{1,s} + q_{2,s} y_{2,s} \right\}
\]

s.t. \( \omega_{1,s} x_1 + x_2 + y_{1,s} \geq 2, \quad \forall s \in S \)

\( \omega_{2,s} x_1 + x_2 + y_{2,s} \geq 3/2, \quad \forall s \in S \)

\( y_{1,s}, y_{2,s} \geq 0, \quad \forall s \in S \)

It seems that through this formulation we could use a decomposition technique: Benders? Dantzig-Wolf? Something more efficient?
Recourse Function

Geometrically,
A Stochastic Generation Expansion Planning example

One-Stage Stochastic Capacity Expansion Planning

Uncertainty in Demand

\[
\min_{G^\text{new}} f = \sum_{j \in \Psi} I_{j} G_{j}^\text{new} + \min_{P_{s,j,m},\text{DNS}_{s,m}} \left\{ \sum_{s \in S} \sum_{j \in \Psi} \sum_{m \in M} \left( \Phi_{s} H_{j} F_{j} P_{s,j,m} \tau_{m} T + p_{s} \text{OM}_{j} P_{s,j,m} \tau_{m} T + p_{s} q \text{DNS}_{s,m} \tau_{m} T \right) \right\}
\]

s.t. \( Cap_{j} = Cap_{j}^\text{old} + G_{j}^\text{new}, \quad \forall j \in \Psi \)

\[
\sum_{j \in \Psi} P_{s,j,m} + \text{DNS}_{s,m} \geq d_{s,m} \left( + R/100 \right), \quad \forall s \in S, \quad \forall m \in M
\]

\[
\sum_{m \in M} P_{s,j,m} \leq A_{j} G_{j}^\text{new}, \quad \forall j \in \Psi, \quad \forall s \in S
\]

\[
\text{DNS}_{s,m} \geq 0, \quad \forall s \in S, \quad \forall m \in M
\]

Load Duration Curve (3 modes)

Something to take care of

Number of variables = \(|M| |S| |\Psi| + 1 + |\Psi|

Number of constraints = \(|\Psi| + (|M| |\Psi| + 2 |M| + |\Psi|) |S|\)
A Stochastic Generation Expansion Planning example

Data

<table>
<thead>
<tr>
<th>Technology</th>
<th>Investment Cost (million $/MW)</th>
<th>O&amp;M Cost ($/MWh)</th>
<th>Heat Rate (MBTU/MWh)</th>
<th>Fuel Cost ($/MBTU)</th>
<th>Availability</th>
<th>Existing Generation (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear</td>
<td>2.475</td>
<td>0.55</td>
<td>10.40</td>
<td>0.82</td>
<td>1.00</td>
<td>100</td>
</tr>
<tr>
<td>Coal</td>
<td>1.534</td>
<td>2.95</td>
<td>9.20</td>
<td>1.54</td>
<td>1.00</td>
<td>450</td>
</tr>
<tr>
<td>NGCC</td>
<td>0.706</td>
<td>2.01</td>
<td>7.50</td>
<td>6.77</td>
<td>1.00</td>
<td>300</td>
</tr>
<tr>
<td>Wind</td>
<td>1.434</td>
<td>2.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.35</td>
<td>40</td>
</tr>
<tr>
<td>Solar</td>
<td>1.695</td>
<td>2.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.35</td>
<td>10</td>
</tr>
</tbody>
</table>

Objective: to obtain the best investment plan for a 30-year period with 12% of required capacity reserve such that satisfies all of the credible scenarios.
## A Stochastic Generation Expansion Planning example

### Demand Scenarios

<table>
<thead>
<tr>
<th>Probability</th>
<th>New Capacity (GW)</th>
<th>Total New Capacity (GW)</th>
<th>Investment (trillion $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Demand</td>
<td>0.25</td>
<td>323.89</td>
<td>1572.86</td>
</tr>
<tr>
<td>Mean Demand</td>
<td>0.5</td>
<td>323.89</td>
<td>2498.60</td>
</tr>
<tr>
<td>Large Demand</td>
<td>0.25</td>
<td>323.89</td>
<td>3424.31</td>
</tr>
<tr>
<td>All in one (Robust)</td>
<td>323.91</td>
<td>323.55</td>
<td>323.7</td>
</tr>
</tbody>
</table>

### Results (uncertainty in demand)

- The “robust” plan is not the same as the Large-Demand case.
- Note: 59 variables and 83 constraints

### Data

<table>
<thead>
<tr>
<th>Technology</th>
<th>Investment Cost (million $/MW)</th>
<th>O&amp;M Cost ($/MWh)</th>
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<th>Fuel Cost ($/MBTU)</th>
<th>Availability</th>
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<td>0.00</td>
<td>0.35</td>
<td>10</td>
</tr>
</tbody>
</table>

### Technology Investment Cost

- **Nuclear**: 2.475 million $/MW
- **Coal**: 1.534 million $/MW
- **NGCC**: 0.706 million $/MW
- **Wind**: 1.434 million $/MW
- **Solar**: 1.695 million $/MW

### O&M Cost ($/MWh)

- Nuclear: 0.55 $/MWh
- Coal: 2.95 $/MWh
- NGCC: 2.01 $/MWh
- Wind: 2.00 $/MWh
- Solar: 2.00 $/MWh

### Heat Rate (MBTU/MWh)

- Nuclear: 10.40 MBTU/MWh
- Coal: 9.20 MBTU/MWh
- NGCC: 7.50 MBTU/MWh
- Wind: 0.00 MBTU/MWh
- Solar: 0.00 MBTU/MWh

### Fuel Cost ($/MBTU)

- Nuclear: 0.82 $/MBTU
- Coal: 1.54 $/MBTU
- NGCC: 6.77 $/MBTU
- Wind: 0.00 $/MBTU
- Solar: 0.00 $/MBTU

### Availability

- Nuclear: 1.00
- Coal: 1.00
- NGCC: 1.00
- Wind: 0.35
- Solar: 0.35

### Existing Generation (MW)

- Nuclear: 100
- Coal: 450
- NGCC: 300
- Wind: 40
- Solar: 10
A Stochastic Generation Expansion Planning example

Uncertainty in Fuel Prices

\[ \min_{G_j^\text{new}} f = \sum_{j \in \Psi} I_j G_j^\text{new} + \min_{P_{s,j,m},\text{DNS}_{s,m}} \left\{ \sum_{s \in S} \sum_{j \in \Psi} \sum_{m \in M} \phi_s H_j F_{s,j} P_{s,j,m} \tau_m T + p_s OM_j P_{s,j,m} \tau_m T + p_s q \text{DNS}_{s,m} \tau_m T \right\} \]

s.t. \[ \text{Cap}_j = \text{Cap}_j^\text{old} + G_j^\text{new}, \quad \forall j \in \Psi \]
\[ \sum_{j \in \Psi} P_{s,j,m} + \text{DNS}_{s,m} \geq d_m (1 + R/100) \quad \forall s \in S, \forall m \in M \]
\[ \sum_{m \in M} P_{s,j,m} \leq A_j G_j^\text{new}, \quad \forall j \in \Psi, \forall s \in S \]
\[ \text{DNS}_{s,m} \geq 0, \quad \forall s \in S, \forall m \in M \]

Fuel Uncertainty

<table>
<thead>
<tr>
<th>Technology</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>0.77</td>
<td>1.54</td>
<td>6.16</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>2.03</td>
<td>6.77</td>
<td>13.54</td>
</tr>
</tbody>
</table>

Results

<table>
<thead>
<tr>
<th>Price Scenarios</th>
<th>Probability</th>
<th>New Capacity (GW)</th>
<th>Total Installed Capacity (GW)</th>
<th>Investment (trillion $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td></td>
<td><strong>Nuclear</strong></td>
<td><strong>Coal</strong></td>
<td><strong>Gas</strong></td>
</tr>
<tr>
<td>Low</td>
<td>0.11</td>
<td>0.004</td>
<td>971.420</td>
<td>323.710</td>
</tr>
<tr>
<td>Low</td>
<td>0.11</td>
<td>0.018</td>
<td>1295.000</td>
<td>0.083</td>
</tr>
<tr>
<td>Low</td>
<td>0.11</td>
<td>0.004</td>
<td>1295.400</td>
<td>0.006</td>
</tr>
<tr>
<td>Medium</td>
<td>0.11</td>
<td>0.031</td>
<td>0.020</td>
<td>647.200</td>
</tr>
<tr>
<td>Medium</td>
<td>0.11</td>
<td><strong>323.890</strong></td>
<td><strong>0.115</strong></td>
<td>323.160</td>
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<tr>
<td>Medium</td>
<td>0.11</td>
<td>323.900</td>
<td>323.550</td>
<td>0.003</td>
</tr>
<tr>
<td>High</td>
<td>0.11</td>
<td>0.096</td>
<td>0.011</td>
<td>647.520</td>
</tr>
<tr>
<td>High</td>
<td>0.11</td>
<td>323.900</td>
<td>0.014</td>
<td>323.260</td>
</tr>
<tr>
<td>High</td>
<td>0.11</td>
<td>646.980</td>
<td>0.118</td>
<td>0.061</td>
</tr>
<tr>
<td>All in One</td>
<td><strong>323.900</strong></td>
<td><strong>0.012</strong></td>
<td><strong>323.250</strong></td>
<td><strong>1851.400</strong></td>
</tr>
</tbody>
</table>

LP with 167 variables and 239 constraints
A Stochastic Generation Expansion Planning example

Uncertainty in Demand and Fuel Prices

\[
\min_{G_j^{new}} f = \sum_{j \in \Psi} I_j G_j^{new} + \min_{p_{s,j,m},DNS_{s,m}} \left\{ \sum_{s \in S} \sum_{j \in \Psi} \sum_{m \in M} \phi_s H_j F_{s,j} P_{s,j,m} r_m T + p_{s} O M_j P_{s,j,m} r_m T + p_{s} q_{DNS_{s,m}} r_m T \right\}
\]

s.t. \( Cap_j = Cap_j^{old} + G_j^{new}, \quad \forall j \in \Psi \)

\[
\sum_{j \in \Psi} P_{s,j,m} + DNS_{s,m} \geq d_{s,m} (1 + R/100) \quad \forall s \in S, \quad \forall m \in M
\]

\[
\sum_{m \in M} P_{s,j,m} \leq A_j G_j^{new}, \quad \forall j \in \Psi, \quad \forall s \in S
\]

\( DNS_{s,m} \geq 0, \quad \forall s \in S, \quad \forall m \in M \)

\( 3^3 = 27 \) Scenarios

### Results (Robust Solution LP with 491 variables and 707 constraints)

<table>
<thead>
<tr>
<th>Random Parameter</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal Price ($/MBTU)</td>
<td>0.77</td>
<td>1.54</td>
<td>6.16</td>
</tr>
<tr>
<td>Natural Gas ($/MBTU)</td>
<td>2.03</td>
<td>6.77</td>
<td>13.54</td>
</tr>
<tr>
<td>Base Demand (MW)</td>
<td>300</td>
<td>600</td>
<td>900</td>
</tr>
</tbody>
</table>

### Comparisons of the three Robust Plans

<table>
<thead>
<tr>
<th></th>
<th>Nuclear</th>
<th>Coal</th>
<th>Gas</th>
<th>Wind</th>
<th>Solar</th>
<th>Total Installed Capacity (GW)</th>
<th>Investment (trillion $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty in Demand</td>
<td>323.91</td>
<td>323.55</td>
<td>323.70</td>
<td>1851.4</td>
<td>0.009</td>
<td>2822.569</td>
<td>4.1814</td>
</tr>
<tr>
<td>Uncertainty in Fuel Prices</td>
<td>323.90</td>
<td>0.012</td>
<td>323.25</td>
<td>1851.4</td>
<td>0.018</td>
<td>2498.580</td>
<td>3.6848</td>
</tr>
<tr>
<td>Uncertainty in Demand and Fuel Prices</td>
<td>647.47</td>
<td>0.008</td>
<td>323.70</td>
<td>1851.3</td>
<td>0.017</td>
<td>2822.495</td>
<td>4.4858</td>
</tr>
</tbody>
</table>
Comments

• The dimension of the DE is highly influenced by handling uncertainty through scenario modeling
• The model might need to be decomposed for realistic applications
• I think that a robust approach is to consider the investment decisions independent of the scenario realizations as much as we can. Generally, we decide whether or not to invest and then observe the realization of uncertain parameters
• For a dynamic version we might want to consider a decision tree for each stage. By doing so, the number of scenarios would be for example $27^T$ for the previous application and $\approx 1.8\times10^{57}$ for 40 1-year periods
• It is recommended to pick the more likely scenarios by sampling. Does it mean a previous treatment of input data before solving the optimization problem? Will we need something like Monte Carlo simulation? Any idea?
Any questions?

Thank you!